# Principles of Mathematical Analysis (PMA) Notes

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# 1 The Real and Complex Number Systems

## 1.1 Introduction

The chapter begins with the discussion of the irrationality of $\sqrt{2}$, namely that there is no rational solution to the equation $p^2 = 2$.

**Definition 1.1.1.** Let $a$ be an integer. We say that $a$ is *even* if there exists an integer $k$ such that $a = 2k$. Similiarly, we say that $a$ is *odd* if there exists an integer $k$ such that $a = 2k + 1$.

**Lemma 1.1.2.** The product of two integers is even if and only if one of them is even.

*Proof.* The proof of this lemma is left to the reader. □

**Theorem 1.1.3** (The Irrationality of $\sqrt{2}$)**.** *There is no rational $p$ such that $p^2 = 2$.*

*Proof.* Suppose, by way of contradiction, there exists $p \in \mathbb{Q}$ such that

$$p^2 = 2 \tag{1}$$

Since $p \in \mathbb{Q}$, then there exists integers $m, n$ such that

$$p = \frac{m}{n}$$

where, without loss of generality, $m$ and $n$ are not both even.

From the following and (1), we see that

$$m^2 = 2n^2. \tag{2}$$

Thus, by definition $m^2$ is even and therefore $m$ is even by Lemma 1.1.2. Then, by definition, there exists a integer $q$ such that $m = 2q$. Therefore, we see that equation (2) becomes

$$m^2 = (2q)^2 = 4q^2 = 2n^2 \tag{3}$$

which implies that

$$n^2 = 2q^2. \tag{4}$$

Furthermore, $n$ must also be even using the same reasoning for $m$. This contradictions our assumption that both $m$ and $n$ are not both even.

Thus, we conclude that there is no rational number $p$ that satisfies the equation $p^2 = 2$ $\square$